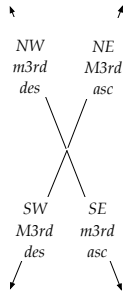


## Harmonic Lattice from D 288 Hz, ratio 1/1 and A at 432 Hz (all notes are in the octave D288Hz–D576Hz, ratios 1/1–2/1)



Notes are given with  
note names  
ratios  
hertz  
& cents + or -

	<b>E</b>	<b>B</b>	<b>F#</b>	<b>C#</b>	<b>G#</b>	<b>D#</b>	<b>A#</b>
	10/9	5/3	5/4	15/8	45/32	135/128	405/256
	320	480	360	540	405	303.75	455.625
	-18	-16	-14	-12	-10	-8	-6
	△	△	△	△	△	△	△

The AM of D288  
and A432 is F#360

(pattern continues) →

<b>Ab</b>	<b>Eb</b>	<b>Bb</b>	<b>F</b>	<b>C</b>	<b>G</b>	<b>D</b>	<b>A</b>	<b>E</b>	<b>B</b>	<b>F#</b>	<b>C#</b>	<b>G#</b>
1024/729	256/243	128/81	32/27	16/9	4/3	1/1 2/1	3/2	9/8	27/16	81/64	243/128	729/512
404.5	303.4	455.11	341.33	512	384	288	432	324	486	364.5	546.75	410.06
-12	-10	-8	-6	-4	-2	0	+2	+4	+6	+8	+10	+12

← (pattern continues)

The HM of D288  
and A432 is F345.6

▽	▽	▽	▽	▽	▽	▽
<b>Gb</b>	<b>Db</b>	<b>Ab</b>	<b>Eb</b>	<b>Bb</b>	<b>F</b>	<b>C</b>
512/405	256/135	64/45	16/15	8/5	6/5	9/5
364.08	546.12	409.6	307.2	460.8	345.6	518.4
+6	+8	+10	+12	+14	+16	+18

The Arithmetic Mean of the octave has a P5th ascending from the root.  
The Harmonic Mean of the octave has a P5th descending from the root.  
(See chart of AM, HM, GM generated from the number ratios of 1, 2, 3, and 4.)

Circles are used for the perfect intervals—the unison, octave, perfect fourth, and perfect fifth.

Ovals are used for the other intervals in the cycle of fourths and fifths from D (those intervals with divisors of 2 and 3, coming from the 3rd harmonic.)

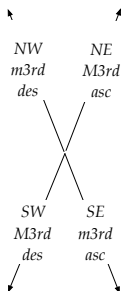
Diamonds are used for notes with divisors of 2, 3, and 5. There are two versions of each note except for G, D, and A.

Notes in brackets are alternatives of “itches with different intentions.” [G#, D#, A#] has almost the same hertz (itches) as (Ab, Eb, Bb), but the note names are different (enharmonic spellings.) Similarly (F#, C#, G#) share almost the same pitches as [Gb, Db, Ab], but with enharmonic spellings, depending on the “intention.”

The Arithmetic mean of the Perfect 5th has a Major 3rd ascending from the root. With the AM pattern repeated, these ascending Major 3rds form their own spine of Perfect 5ths.

The Harmonic mean of the Perfect 5th has a Major 3rd descending from the 5th. With the HM pattern repeated, these descending Major 3rds form their own spine of Perfect 5ths

## Harmonic Lattice from D 293.334 Hz, ratio 1/1 and A at 440 Hz\* (all notes are in the octave D293.34–D586.667, ratios 1/1–2/1)



Notes are given with  
note names  
ratios  
hertz  
& cents + or -

	<b>E</b>	<b>B</b>	<b>F#</b>	<b>C#</b>	<b>G#</b>	<b>D#</b>	<b>A#</b>
	10/9	5/3	5/4	15/8	45/32	135/128	405/256
	325.93	488.89	366.67	550	412.5	309.375	464.06
	-18	-16	-14	-12	-10	-8	-6
	△	△	△	△	△	△	△

The AM of D293.334 and A440 is F#366.67 (pattern continues) →

<b>Ab</b>	<b>Eb</b>	<b>Bb</b>	<b>F</b>	<b>C</b>	<b>G</b>	<b>D</b>	<b>A</b>	<b>E</b>	<b>B</b>	<b>F#</b>	<b>C#</b>	<b>G#</b>
1024/729	256/243	128/81	32/27	16/9	4/3	1/1 2/1	3/2	9/8	27/16	81/64	243/128	729/512
412.04	309.02	463.54	347.655	521.482	391.11	293.334	440	330	495	371.25	556.88	417.66
-12	-10	-8	-6	-4	-2	0	+2	+4	+6	+8	+10	+12

← (pattern continues) The HM of D293.334 and A440 is F352

▽	▽	▽	▽	▽	▽	▽
<b>Gb</b>	<b>Db</b>	<b>Ab</b>	<b>Eb</b>	<b>Bb</b>	<b>F</b>	<b>C</b>
512/405	256/135	64/45	16/15	8/5	6/5	9/5
370.83	556.25	417.186	312.89	469.33	352	528
+6	+8	+10	+12	+14	+16	+18

The Arithmetic Mean of the octave has a P5th ascending from the root.  
The Harmonic Mean of the octave has a P5th descending from the root.  
(See chart of AM, HM, GM generated from the number ratios of 1, 2, 3, and 4.)

The Arithmetic mean of the Perfect 5th has a Major 3rd ascending from the root. With the AM pattern repeated, these ascending Major 3rds form their own spine of Perfect 5ths.  
The Harmonic mean of the Perfect 5th has a Major 3rd descending from the 5th. With the HM pattern repeated, these descending Major 3rds form their own spine of Perfect 5ths

Circles are used for the perfect intervals—the unison, octave, perfect fourth, and perfect fifth.

Ovals are used for the other intervals in the cycle of fourths and fifths from D (those intervals with divisors of 2 and 3, coming from the 3rd harmonic.)

Diamonds are used for notes with divisors of 2, 3, and 5. There are two versions of each note except for G, D, and A.

Notes in brackets are alternatives of “pitches with different intentions.” [G#, D#, A#] has almost the same hertz (pitches) as (Ab, Eb, Bb), but the note names are different (enharmonic spellings.) Similarly (F#, C#, G#) share almost the same pitches as [Gb, Db, Ab], but with enharmonic spellings, depending on the “intention.”

\* Since A440 is the conventional standard base pitch, I have included this page showing hertz values using A440 rather than A432. Note that the ratios and cents are still the same and are calculated from D ratio 1/1 and octave 2/1.